



PII: S0017-9310(97)00002-1

# Heat and mass transfer in film absorption in the presence of non-absorbable gases

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(Received 8 October 1995 and in final form 5 December 1996)

**Abstract**—This paper is concerned with the effect of non-absorbable gases on the combined heat and mass transfer process in film absorption. Some simple models of the phenomenon have been proposed before which have under-predicted the mass flux. Here, a detailed theoretical model has been developed for the film, taking into consideration bulk movements in the vapor and liquid phases in the direction perpendicular to the film flow, which occur to compensate for the molal flux due to concentration gradients in the absorbent and vapor. The energy and diffusion equations were solved analytically for the entrance region in both the liquid and gas phases to give the temperature and concentration distributions, from which heat and mass fluxes may be determined. The reduction in mass flux due to non-absorbable gases is related to the need for the absorbate to diffuse through a thin layer of these gases blocking its access to the interface, before it can be absorbed. This reduction was found to depend most critically on the ratio of resistance to absorbate diffusion between the liquid and the gas. Even a small amount of non-absorbables can inhibit absorption significantly. Particular cases including isothermal absorption have been considered and the effect of the heat of absorption and other process parameters has been studied. © 1997 Elsevier Science Ltd.

## 1. INTRODUCTION

The presence of non-absorbable gases in absorption systems has long been known to have a deleterious effect on their performance. In closed-cycle systems, small quantities of such gases originate from corrosion or minor leaks; in open-cycle systems, rather large quantities are introduced continuously. It has been observed that even small amounts of such gases lead to a significant reduction in the absorption rate. The non-absorbables tend to accumulate at the interface between the absorbate and absorbent and block the access of the former to the latter. The absorbate vapor must then diffuse through a layer of gas which adds resistance to the process. It is common practice in operating absorbers to purge out the non-absorbable gases.

The influence of non-absorbables in absorption processes is equivalent in some respects to that of “non-condensables” in the more familiar field of condensation which reduce the transfer rate by inhibiting access of the condensing vapor to the cold surface. However, there are important differences between the two processes. In absorption, the liquid film contains not only the absorbate, but also the absorbent and offers a resistance not only to heat, but also to mass transfer. The absorbate must diffuse not only through the gas, but also through the liquid layer. Furthermore, the equilibrium condition at the gas-liquid interface in absorption is not only a saturation pressure-temperature relation as in condensation, but also depends on the interfacial composition of the absorb-

ent solution which in turn depends on the mass transfer.

The combined heat and mass transfer process in absorption in the absence of non-absorbables was the subject of several studies in recent years. Absorption in laminar falling films was modeled and analyzed by Nakoryakov and Grigor'eva [1, 2], Yih and Seagrave [3] and Grossman [4]. An improvement to these models, for cases where the absorbate concentration is comparable to that of the absorbent, was proposed by Brauner [5] and Brauner *et al.* [6]. Andberg and Vliet [7] modeled the absorption in a laminar film around a horizontal circular tube. Grossman and Heath [8] extended the falling film analysis to turbulent flow. Experimental data for absorption accompanied by a significant heat effect is unfortunately rather scarce; however, the more limited case of isothermal absorption was studied extensively for a wide range of Reynolds numbers [9].

Condensation heat transfer in the presence of non-condensables was the subject of numerous investigations. Analytical models for prediction of condensation rates of a one-component vapor were developed by Sparrow and Lin [10], Minkowycz and Sparrow [11] and Rose [12]. Denny *et al.* [13] extended the model of Ref. [11] by taking into consideration forced flow downward of the vapor/non-condensable mixture. Experimental results were reported by Henderson and Marchello [14], Slegers and Seban [15] and Al-Diwany and Rose [16] on the condensation of steam mixed with air and other non-condensable gases. Condensation of binary and

## NOMENCLATURE

$A$	dimensionless constant in equations (16)–(17)	$v$	bulk velocity in the $y$ -direction near interface ( $\text{m s}^{-1}$ )
$A_o$	value of $A$ for the case of pure isothermal absorption free of non-absorbables	$W$	number of moles of absorbate per unit volume of the gas phase ( $\text{kmole m}^{-3}$ )
$B$	dimensionless constant in equations (18)–(19)	$W_G$	total number of moles per unit volume of the gas phase, $W_{\text{III}} + W$ ( $\text{kmole m}^{-3}$ )
$C$	concentration of absorbate in the liquid solution ( $\text{kmole m}^{-3}$ )	$W_{\text{III}}$	number of moles of non-absorbable gas (substance III) per unit volume of the gas phase ( $\text{kmole m}^{-3}$ )
$C_e$	equilibrium concentration of solution at temperature $T_o$ with vapor pressure $P_e$ ( $\text{kmole m}^{-3}$ )	$x$	coordinate in direction of flow (m)
$C_I$	concentration of absorbent (substance I) in liquid solution ( $\text{kmole m}^{-3}$ )	$y$	coordinate in direction perpendicular to flow (m).
$C_L$	total number of moles per unit volume of liquid, $C_I + C$ ( $\text{kmole m}^{-3}$ )	Greek symbols	
$D$	diffusion coefficient of absorbate ( $\text{m}^2 \text{s}^{-1}$ )	$\alpha$	thermal diffusivity ( $\text{m}^2 \text{s}^{-1}$ )
$h_M$	mass transfer coefficient from interface to bulk of solution ( $\text{m s}^{-1}$ )	$\beta$	dimensionless gas–liquid thermal property ratio, equation (26b)
$\bar{H}_a$	heat of absorption of absorbate in solution ( $\text{kJ kmole}^{-1}$ )	$\gamma$	dimensionless concentration of absorbate in the liquid, equation (7)
$j$	molal diffusive flux of the absorbate ( $\text{kmole m}^{-2} \text{s}^{-1}$ )	$\gamma_L$	dimensionless liquid diffusion resistance parameter, equation (14)
$k$	thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ )	$\eta$	dimensionless similarity coordinate, equation (15)
$K_{1-4}$	dimensionless constants of integration, equations (21)–(24)	$\theta$	dimensionless liquid temperature, equation (7)
$Le$	Lewis number, $D/\alpha$	$\theta_o$	dimensionless gas–liquid temperature ratio, equation (26a)
$N$	molal flux of absorbate in the $y$ -direction in the solution ( $\text{kmole s}^{-1} \text{m}^{-2}$ )	$\Lambda$	dimensionless heat of absorption, equation (26b)
$N_{io}$	molal flux of absorbate in the $y$ -direction at the interface, for the case of pure isothermal absorption free of non-absorbables ( $\text{kmole s}^{-1} \text{m}^{-2}$ )	$\xi$	dimensionless similarity coordinate, equation (15)
$N_I$	molal flux of absorbent in the $y$ -direction in the solution ( $\text{kmole s}^{-1} \text{m}^{-2}$ )	$\tau$	dimensionless gas temperature, equation (7)
$P$	vapor pressure ( $\text{N m}^{-2}$ )	$\phi$	dimensionless gas–liquid diffusion resistance ratio, equation (26b)
$P_e$	total pressure in the gas phase ( $\text{N m}^{-2}$ )	$\omega$	dimensionless concentration of absorbate in the gas, equation (7)
$P_o$	equilibrium vapor pressure of liquid solution at temperature $T_o$ and concentration $C_o$ ( $\text{N m}^{-2}$ )	$\omega_{\text{III}}$	dimensionless bulk mole fraction of non-absorbables, equation (26a).
$Pe$	Peclet number, $u_o x/D$ (dimensionless)	Subscripts	
$R$	universal gas constant ( $\text{kJ kmole}^{-1} \text{K}^{-1}$ )	L	liquid solution
$Sh_x$	Sherwood number, $h_M x/D$ (dimensionless)	G	gas phase
$T$	temperature (K)	i	at interface
$T_e$	equilibrium temperature of solution at concentration $C_o$ with vapor at pressure $P_e$ (K)	I	substance I (absorbent)
$u_o$	flow velocity in the $x$ -direction near interface ( $\text{m s}^{-1}$ )	II	substance II (absorbate)
		III	substance III (non-absorbable gas)
		o	outside boundary layers (initial) in the liquid phase
		$\infty$	outside boundary layers (initial) in the gas phase.

multicomponent mixtures in the presence of non-condensables was treated by Taitel and Tamir [17], Lee and Rose [18] and Kotake [19].

Only a few studies have been reported in the literature on the effect of non-absorbable gases on the heat and mass transfer process in film absorption. Experi-

ments by Burdukov *et al.* [20] have indicated a significant reduction in the absorption rate of water vapor in aqueous lithium bromide solution flowing down a tube bundle, due to the presence of small amounts of non-absorbables. Yang and Wood [21] found a smaller reduction of the absorption rate in more extensive experiments with wavy-laminar films of aqueous lithium chloride. Related experimental and analytical studies have been conducted by Vliet and co-workers [22, 23]. In an attempt at a quantitative understanding of the process, a first model of film absorption in the presence of non-absorbable gases was developed [24] where the scaling parameters of the problem were determined and an equilibrium relation was derived between vapor pressure, concentration and temperature at the gas-liquid interface. The main shortcoming of this model, which was improved somewhat later [25], was in not taking into consideration adequately bulk movements in the vapor and liquid phases in the direction perpendicular to the film flow, which occur in order to compensate for the molal flux due to concentration gradients in the liquid and vapor. Habib *et al.* [26] have extended this model to include the effect of the transverse velocity component in the vapor phase, but not in the liquid phase.

In this study, a detailed and more complete model is developed to account for these and other phenomena in examining the effect of non-absorbable gases on the combined heat and mass transfer process in film absorption. The model is based on the conservation laws and thermodynamic equilibrium principles, for the case of an absorbent liquid film in contact with a gaseous mixture of an absorbate and non-absorbables.

## 2. MODEL AND EQUATIONS

Figure 1 describes schematically the system under consideration. A film of liquid solution composed of substances I (absorbent) and II (absorbate), flows down over an inclined surface. The film is in contact with a gas mixture composed of the absorbate vapor and of substance III (non-absorbable gas), at a total pressure  $P_e$ . Substance I is contained only in the liquid phase and substance III only in the gas phase; substance II is transferred between the gas and the liquid. As a practical example to this system, one may consider a film of an aqueous solution of LiBr (substance I) flowing over the surface of an absorber tube, in contact with water vapor (substance II) containing a certain amount of air (substance III). At  $x = 0$ , the liquid and the gas are at uniform temperatures  $T_0$  and  $T_\infty$ , respectively, and the concentrations of the absorbate in the liquid and gas are  $C_0$  and  $W_\infty$ , respectively. The initial absorbate vapor pressure in the solution  $P_0(C_0, T_0)$  is lower than  $P_e$ . This results in absorption taking place at the gas-liquid interface, with substance II diffusing through the gas phase and into the liquid film; the heat generated in the absorption produces simultaneous heat transfer.

The following assumptions have been made in formulating the model: (1) the physical properties of the liquid and gas are constant and independent of temperature and concentration; (2) the mass of vapor absorbed per unit time is small compared to the film flow rate and does not affect it; (3) natural convection is negligible; (4) there is no slip between the gas and the liquid that share a common velocity  $u_0$  at the interface. This flow velocity is sufficiently low not to

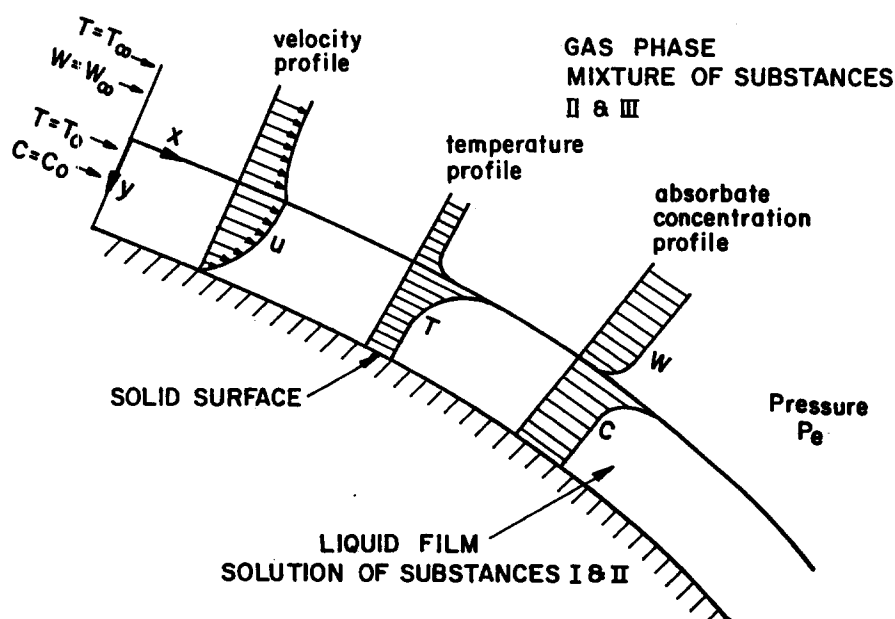


Fig. 1. Schematic description of a falling film with typical temperature, concentration and velocity profiles.

disturb the interface; (5) thermodynamic equilibrium exists at the interface between the absorbate contained in the gas and in the liquid; (6) thermal diffusion, diffusion-induced thermo-effects and interdiffusion are negligible; (7) the velocity over the thermal and diffusion boundary layers thickness is practically uniform:  $u = u_0$ ; (8) the total molal concentrations in both the liquid and gas phases,  $C_L = C_1 + C$  and  $W_G = W_{III} + W$ , respectively, are constant. This assumption allows the use of Fick's law in the form:

$$j_{II} = -D_G \frac{\partial W}{\partial y} = -D_L \frac{\partial C}{\partial y}. \quad (1)$$

Under the above assumptions, the combined heat and mass transfer process in the system at steady state conditions is governed by the diffusion and energy equations.

$$u_0 \frac{\partial C}{\partial x} + v_L \frac{\partial C}{\partial y} = D_L \frac{\partial^2 C}{\partial y^2} \quad (2)$$

$$u_0 \frac{\partial T}{\partial x} + v_L \frac{\partial T}{\partial y} = \alpha_L \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u_0 \frac{\partial W}{\partial x} + v_G \frac{\partial W}{\partial y} = D_G \frac{\partial^2 W}{\partial y^2} \quad (4)$$

$$u_0 \frac{\partial T}{\partial x} + v_G \frac{\partial T}{\partial y} = \alpha_G \frac{\partial^2 T}{\partial y^2} \quad (5)$$

where  $v_L$  and  $v_G$  are bulk velocities in the  $y$ -direction in the liquid and vapor phases, respectively; these velocities are functions of  $x$  only (under assumption (7) and continuity).

Boundary conditions at the interface ( $y=0$ ) require temperature and absorbate vapor pressure equilibrium between the gas and liquid phases, and a match between the heat and mass fluxes. These conditions, in the above order, may be expressed as follows:

$$T_L = T_G \quad (6a)$$

$$\begin{aligned} & \ln\left(\frac{P}{P_e}\right) \\ &= \left[ \frac{T(C_e - C_o) + C(T_e - T_o) + (T_o C_o - T_e C_e)}{(T_e - T_o)(C_e - C_o)} \right] \\ & \quad \times \ln\left(\frac{P_e}{P_o}\right) \end{aligned} \quad (6b)$$

$$-k_L \left(\frac{\partial T}{\partial y}\right)_L + k_G \left(\frac{\partial T}{\partial y}\right)_G = H_a \left[ C v_L - D_L \frac{\partial C}{\partial y} \right] \quad (6c)$$

$$C_1 v_L + D_L \frac{\partial C}{\partial y} = 0 \quad (6d)$$

$$W_{III} v_G + D_G \frac{\partial W}{\partial y} = 0 \quad (6e)$$

$$C v_L - D_L \frac{\partial C}{\partial y} = W v_G - D_G \frac{\partial W}{\partial y}. \quad (6f)$$

Equation (6b) expresses the interfacial equilibrium relation which depends on the thermodynamic properties of the particular fluids at hand. A pressure-temperature-concentration relation typical of absorption fluids is employed [25]. Equations (6d) and (6e) represent the impermeability of the interface to substances I and III.

Figure 1 describes some typical profiles of velocity, temperature and concentration in the system. Starting from  $x=0$ , momentum, thermal and concentration boundary layers begin to develop in the liquid and in the gas, until those in the liquid fill the entire film thickness. The rate of development varies for the different boundary layers depending on their respective diffusivities. In this work we will consider only the region with developing boundary layers, for which an analytical solution in closed form may be obtained. The analysis may be extended to the fully-developed region by solving the equations numerically [4]. The physical behavior of the absorbing film is represented quite well by the present case.

### 3. SOLUTION

It is convenient to introduce, into the equations, a dimensionless form of the temperature and concentration:

$$\begin{aligned} \gamma &= \frac{C - C_o}{C_e - C_o}; \quad \theta = \frac{T - T_o}{T_e - T_o}; \\ \omega &= \frac{W - W_\infty}{W_G - W_\infty}; \quad \tau = \frac{T - T_\infty}{T_e - T_\infty}. \end{aligned} \quad (7)$$

Using the dimensionless variables and applying the boundary conditions (6d) and (6e), we obtain from equations (2)–(5):

$$u_0 \frac{\partial \gamma}{\partial x} = D_L \left[ \frac{\partial^2 \gamma}{\partial y^2} + \left( \frac{C_e - C_o}{C_1} \frac{\partial \gamma}{\partial y} \right)_{y=0} \frac{\partial \gamma}{\partial y} \right] \quad (8)$$

$$u_0 \frac{\partial \theta}{\partial x} = \alpha_L \left[ \frac{\partial^2 \theta}{\partial y^2} + \frac{D_L}{\alpha_L} \left( \frac{C_e - C_o}{C_1} \frac{\partial \gamma}{\partial y} \right)_{y=0} \frac{\partial \theta}{\partial y} \right] \quad (9)$$

$$u_0 \frac{\partial \omega}{\partial x} = D_G \left[ \frac{\partial^2 \omega}{\partial y^2} + \left( \frac{W_G - W_\infty}{W_{III}} \frac{\partial \omega}{\partial y} \right)_{y=0} \frac{\partial \omega}{\partial y} \right] \quad (10)$$

$$u_0 \frac{\partial \tau}{\partial x} = \alpha_G \left[ \frac{\partial^2 \tau}{\partial y^2} + \frac{D_G}{\alpha_G} \left( \frac{W_G - W_\infty}{W_{III}} \frac{\partial \omega}{\partial y} \right)_{y=0} \frac{\partial \tau}{\partial y} \right] \quad (11)$$

with the boundary conditions:

$$\gamma = \theta = 0 \quad @ \quad x = 0 \quad \& \quad y \rightarrow \infty \quad (12a)$$

$$\omega = \tau = 0 \quad @ \quad x = 0 \quad \& \quad y \rightarrow -\infty \quad (12b)$$

and the boundary conditions (6) at the interface ( $y = 0$ ) become:

$$\theta(T_e - T_o) + T_o = \tau(T_e - T_\infty) + T_\infty \quad (13a)$$

$$\ln \left[ 1 + (\omega - 1) \left( \frac{W_G - W_\infty}{W_G} \right) \right] = (\theta + \gamma - 1) \ln \left( \frac{P_e}{P_o} \right) \quad (13b)$$

$$\frac{\partial \theta}{\partial y} - \frac{k_G (T_e - T_\infty)}{k_L (T_e - T_o)} \frac{\partial \tau}{\partial y} = \frac{\bar{H}_a D_L (C_e - C_o)}{k_L (T_e - T_o)} \frac{\partial \gamma}{\partial y} \left[ 1 + \frac{\gamma(C_e - C_o) + C_o}{C_1} \right] \quad (13c)$$

$$\frac{\partial \gamma}{\partial y} \left( 1 + \frac{\gamma(C_e - C_o) + C_o}{C_1} \right) = \frac{D_G (W_G - W_\infty)}{D_L (C_e - C_o)} \frac{\partial \omega}{\partial y} \times \left( 1 + \frac{\omega(W_G - W_\infty) + W_\infty}{W_{III}} \right) \quad (13d)$$

In deriving equation (13b) from equation (6b), we have assumed Dalton's law to apply in the gas phase at the interface (which is at a uniform temperature) and, hence,  $P/P_o = W/W_G$ . The values of  $C_1$ , in equations (8)–(9) and  $W_{III}$  in equations (10)–(11), are not constant. By applying assumption (8) and equation (7) we can define the following relations:

$$\frac{C_1}{C_e - C_o} = \gamma_L - \gamma \quad \text{and} \quad \frac{W_{III}}{W_G - W_\infty} = 1 - \omega$$

where  $\gamma_L = \frac{C_L - C_o}{C_e - C_o}$ . (14)

In order to solve the governing equations with their boundary conditions we introduce similarity variables in the following form:

$$\xi_L = \frac{y}{2\sqrt{x D_L / u_o}}; \quad \xi_G = \frac{y}{2\sqrt{x D_G / u_o}};$$

$$\eta_L = \frac{y}{2\sqrt{x \alpha_L / u_o}}; \quad \eta_G = \frac{y}{2\sqrt{x \alpha_G / u_o}}. \quad (15)$$

Using equation (14) and the new similarity variables, equations (8)–(11) become:

$$\frac{d^2 \gamma}{d\xi_L^2} + 2(\xi_L - A) \frac{d\gamma}{d\xi_L} = 0 \quad (16)$$

$$\frac{d^2 \theta}{d\eta_L^2} + 2\left(\eta_L - A \sqrt{\frac{D_L}{\alpha_L}}\right) \frac{d\theta}{d\eta_L} = 0 \quad (17)$$

$$\frac{d^2 \omega}{d\xi_G^2} + 2(\xi_G - B) \frac{d\omega}{d\xi_G} = 0 \quad (18)$$

$$\frac{d^2 \tau}{d\eta_G^2} + 2\left(\eta_G - B \sqrt{\frac{D_G}{\alpha_G}}\right) \frac{d\tau}{d\eta_G} = 0 \quad (19)$$

where

$$A = \left( \frac{1}{2(\gamma - \gamma_L)} \frac{d\gamma}{d\xi_L} \right)_{\xi_L=0} = \text{constant} \quad (20a)$$

$$B = \left( \frac{1}{2(\omega - 1)} \frac{d\omega}{d\xi_G} \right)_{\xi_G=0} = \text{constant}. \quad (20b)$$

Integrating equations (16)–(19) and applying the boundary conditions (12) the following expressions are obtained for the dimensionless concentrations and temperatures:

$$\gamma = K_1 [1 - \text{erf}(\xi_L - A)] \quad \xi_L \geq 0 \quad (21)$$

$$\theta = K_2 \left[ 1 - \text{erf} \left( \eta_L - A \sqrt{\frac{D_L}{\alpha_L}} \right) \right] \quad \eta_L \geq 0 \quad (22)$$

$$\omega = K_3 [1 + \text{erf}(\xi_G - B)] \quad \xi_G \leq 0 \quad (23)$$

$$\tau = K_4 \left[ 1 + \text{erf} \left( \eta_G - B \sqrt{\frac{D_G}{\alpha_G}} \right) \right] \quad \eta_G \leq 0 \quad (24)$$

where  $K_1, K_2, K_3$  and  $K_4$  are constants of integration still to be determined. Substituting equations (21)–(24) into the boundary conditions (13) yields:

$$K_2 [1 + \text{erf}(A \sqrt{Le_L})] = \theta_o K_4 [1 - \text{erf}(B \sqrt{Le_G})] + (1 - \theta_o) \quad (25a)$$

$$\ln \{ 1 + \omega_{III} [K_3 (1 - \text{erf}(B)) - 1] \} = \{ K_1 [1 + \text{erf}(A)] + K_2 [1 + \text{erf}(A \sqrt{Le_L})] - 1 \} \times \ln \left( \frac{P_e}{P_o} \right) \quad (25b)$$

$$K_2 \exp(-A^2 Le_L) + \beta K_4 \theta_o \exp(-B^2 Le_G) = \frac{A \sqrt{\pi}}{\sqrt{Le_L}} \Lambda \quad (25c)$$

$$B = \phi A \quad (25d)$$

where

$$\theta_o = \frac{T_e - T_\infty}{T_e - T_o} \quad \omega_{III} = \frac{W_G - W_\infty}{W_G} \quad (26a)$$

$$\Lambda = \frac{\bar{H}_a D_L C_L}{k_L (T_e - T_o)} \quad \phi = \frac{C_L}{W_G} \sqrt{\frac{D_L}{D_G}} \quad \beta = \frac{k_G \sqrt{\alpha_L}}{k_L \sqrt{\alpha_G}}. \quad (26b)$$

The constants  $A$  and  $B$ , in equation (20), may also be evaluated:

$$A = \left( \frac{1}{2(\gamma - \gamma_L)} \frac{d\gamma}{d\xi_L} \right)_{\xi_L=0} = \frac{K_1 \exp(-A^2)}{\sqrt{\pi} [\gamma_L - K_1 (1 + \text{erf}(A))]} = \frac{K_1 \exp(-A^2)}{\sqrt{\pi} (\gamma_L - \gamma_i)} \quad (27a)$$

$$B = \left( \frac{1}{2(\omega-1)} \frac{d\omega}{d\xi_G} \right)_{\xi_G=0} = \frac{K_3 \exp(-B^2)}{\sqrt{\pi[K_3(1 - \operatorname{erf}(B)) - 1]}} = \frac{K_3 \exp(-B^2)}{\sqrt{\pi(\omega_i - 1)}}. \quad (27b)$$

The set (25) provides four algebraic equations for the four unknowns  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  in terms of the characteristic parameters of the problem and the constants  $A$  and  $B$ , which are themselves functions of these parameters (equation (27)). As evident from equation (25), there is a total of nine dimensionless parameters characterizing the problem under the most general case. These include the Lewis numbers in the liquid and gas phases ( $Le_L$ ,  $Le_G$ ), the pressure ratio ( $P_e/P_o$ ), the dimensionless liquid concentration  $\gamma_L$  and the five parameters defined in equation (26). Note that  $\beta$  expresses a ratio of thermal properties between the gas and the liquid;  $\Lambda$  is a dimensionless heat of absorption already encountered in film absorption problems [4–8], which in this case appears in conjunction with the liquid Lewis number;  $\phi$  expresses a diffusion resistance ratio between the gas and the liquid;  $\omega_{III}$  is the mole fraction of non-absorbable gases in the bulk of the gas stream, away from the interface;  $\theta_o$  expresses a dimensionless temperature ratio between the gas and the liquid.

The results for the temperature and concentration distributions make it possible to calculate the heat and molal fluxes and coefficients and, hence, the Nusselt and Sherwood numbers. The molal flux of absorbate in the  $y$ -direction at the interface is given by:

$$N_i = \left[ C v_L - D_L \frac{\partial C}{\partial y} \right]_{y=0} = A C_L \sqrt{\frac{u_o D_L}{x}}. \quad (28)$$

The corresponding Sherwood number, based on the length  $x$ , is:

$$\begin{aligned} Sh_x &= \frac{h_M x}{D_L} \\ &= \frac{x}{D_L} \frac{N_i}{C(y=0) - C_o} \\ &= \frac{A}{K_1 [1 + \operatorname{erf}(A)]} \frac{C_L \sqrt{Pe_L}}{C_e - C_o} \end{aligned} \quad (29)$$

where  $Pe_L = u_o x / D_L$  is the liquid Peclet number.

It can be shown that the ratio between the velocities in the liquid in the  $x$  and  $y$  directions is given by:

$$\frac{v_L}{u_o} = \frac{A}{\sqrt{Pe_L}}. \quad (30)$$

As one can see from equation (30), neglecting the velocities in the  $y$  direction, as has been done in earlier models, may result in a large error in the solution for the entrance region where  $Pe_L$  is very small.

It is evident that the Sherwood number is a function of  $K_1$  and  $A$  which depend on the characteristic dimensionless parameters of the problem, including the con-

centration of non-absorbables. The effect of the latter is in reducing both the mass flux and the concentration difference driving force. This reduction may be expressed by the ratio of the actual mass flux ( $N_i$ ) to that in the case of pure isothermal absorption, free of non-absorbables ( $N_{io}$ ). There, the driving force is at the maximum possible value under the given conditions and is equal to  $(C_e - C_o)$ . We thus obtain:

$$\frac{N_i}{N_{io}} = \frac{A}{A_o}. \quad (31)$$

Expressions similar to equations (28) and (29) may be obtained for the heat flux and Nusselt number. In most cases of practical interest it is possible to neglect the heat flux from the interface to the gas compared to that to the liquid; the heat flux is then simply equal to the mass flux times the heat of absorption.

#### 4. RESULTS AND DISCUSSION

The solution of the governing equations has yielded the temperature and concentration distributions in the liquid and gas phases (equations (21)–(24)), with the coefficients  $K_1$ ,  $K_2$ ,  $K_3$  and  $K_4$  determined from the four algebraic equations (25). The behavior in the most general case is quite complex and depends, as mentioned earlier, on nine dimensionless parameters characterizing the problem. In discussing the results we will consider particular cases of interest, starting with the simplest condition and proceeding to more complex ones.

##### 4.1. Case 1

Let us consider first the simplest case of pure isothermal absorption, free of non-absorbable gases and of any heat effects. This condition had been analyzed by Higbie [27] followed by Emmert and Pigford [28] (without considering, however, the effect of the transverse velocity component  $v$ ) and formed the basis for the well-known penetration theory. Here, the mass flux is limited only by the diffusion resistance on the liquid side. Thus, out of the governing equations only equation (2) applies, for the concentration distribution in the liquid, with the equilibrium boundary condition (6b) at the interface. This boundary condition reduces in this case to:  $C = C_e$  at  $y = 0$ . Our general solution reduces to:

$$y = \frac{1 - \operatorname{erf}\left(\frac{y}{2\sqrt{x D_L / u_o}} - A_o\right)}{1 + \operatorname{erf}(A_o)} \quad (32)$$

where  $A_o$  is determined from equation (27a):

$$A_o \sqrt{\pi(\gamma_L - 1)} [1 + \operatorname{erf}(A_o)] = \exp(-A_o^2). \quad (33)$$

In this case, the absorption process is controlled by one parameter only,  $\gamma_L$ , the reciprocal of which is

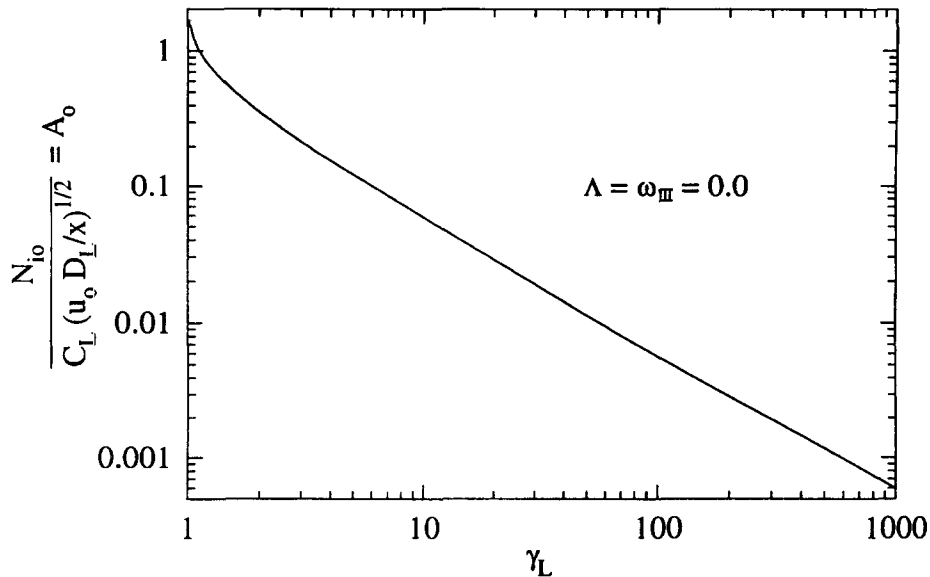


Fig. 2. Dimensionless mass flux in pure isothermal absorption as a function of  $\gamma_L$ —the dimensionless liquid diffusion resistance parameter.

proportional to the concentration driving force for diffusion in the liquid. Figure 2 describes the mass flux  $N_{i0}$  (represented in dimensionless form by the value of the constant  $A_0$ , per equation (28)) as a function of  $\gamma_L$  under these conditions of pure isothermal absorption ( $\Lambda = \omega_{III} = 0$ ). As expected, the mass flux decreases with increasing  $\gamma_L$ , which indicates a decreasing driving force ( $C_e - C_0$ ). For large  $\gamma_L$  and corresponding values of  $A_0$  much smaller than unity, the relation becomes almost linear on the logarithmic scale, as evident from Fig. 2, and may be expressed by a simplified version of equation (33):

$$A_0 \approx \frac{1}{\sqrt{\pi(\gamma_L - 1)}}. \quad (34)$$

In the other extreme of small  $\gamma_L$  ( $\gamma_L$  approaching unity),  $A_0$  becomes very large. This condition, where  $C_e$  approaches  $C_L$ , represents a vanishing resistance to mass transfer. The absence of non-absorbables and heat effects makes for the most favorable conditions for absorption. The pure isothermal absorption case and the associated mass flux  $N_{i0}$  will be used as the reference condition for comparison to other more complex cases.

#### 4.2. Case 2

Next, let us consider the case of non-isothermal absorption free of non-absorbable gases. Here, the mass flux is limited not only by the resistance to diffusion on the liquid side, but also by the heat effect created by the absorption at the interface, which raises the temperature and, hence, the vapor pressure of the absorbate and inhibits the process. This case has been analyzed by Grigor'eva and Nakoryakov [2] neglecting, however, the influence of the transverse velocity component. Of the governing equations for the gen-

eral case, only equations (2) and (3) apply here for the temperature and concentration fields in the liquid, with the boundary conditions (6b), (6c) and (6d) at the interface. The equilibrium boundary condition in its dimensionless form (13b) reduces to:  $\theta + \gamma = 1$  at  $y = 0$ . The solution to equations (2) and (3) is given by equations (21) and (22) with the constants  $K_1$  and  $K_2$  calculated from reduced versions of equations (25b) and (25c) as follows:

$$K_1[1 + \operatorname{erf}(A)] + K_2[1 + \operatorname{erf}(A\sqrt{Le_L})] = 1 \quad (35a)$$

$$K_2 \exp(-A^2 Le_L) = \frac{A\sqrt{\pi}}{\sqrt{Le_L}} \Lambda. \quad (35b)$$

In this case the absorption process is controlled by three parameters. In addition to  $\gamma_L$ , the liquid diffusion resistance parameter, there is also the liquid Lewis number  $Le_L$  and the dimensionless heat of absorption  $\Lambda$ . Figure 3 describes the interfacial temperature and concentration ( $\theta_i$  and  $\gamma_i$  derived from equations (21) and (22)) and Fig. 4—the reduced molal flux  $N_i/N_{i0}$ , as functions of  $\Lambda$ , for different values of  $Le_L$  and for a typical value of  $\gamma_L = 2.5$ . These results have been obtained from the solution for the general case by substituting:  $\beta = \omega_{III} = 0$  and confirmed independently by solving the governing equations for this specific case. It is evident that the heat effect causes an increase in the interfacial temperature accompanied by a reduction in interfacial concentration and a significant reduction in the mass flux compared to the isothermal case. The reduction is greater for smaller values of the liquid Lewis number, which represents greater resistance to diffusion. The mass flux decreases with  $\Lambda$  and reduces, as expected,

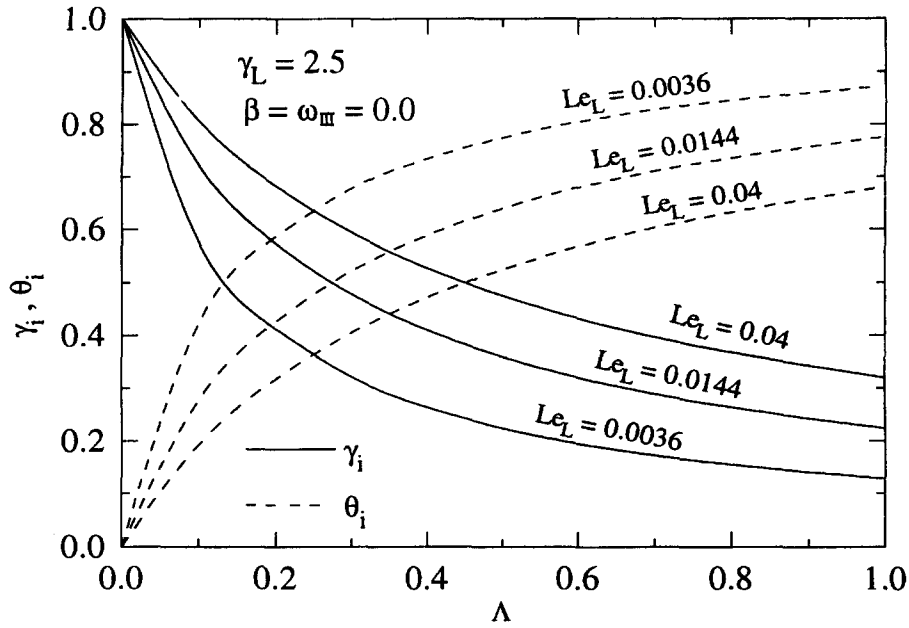


Fig. 3. Non-isothermal absorption in the absence of non-absorbables: interfacial temperature ( $\theta_i$ ) and concentration ( $\gamma_i$ ) as function of the dimensionless heat of absorption for different values of the liquid Lewis number ( $\gamma_L = 2.5$ ).

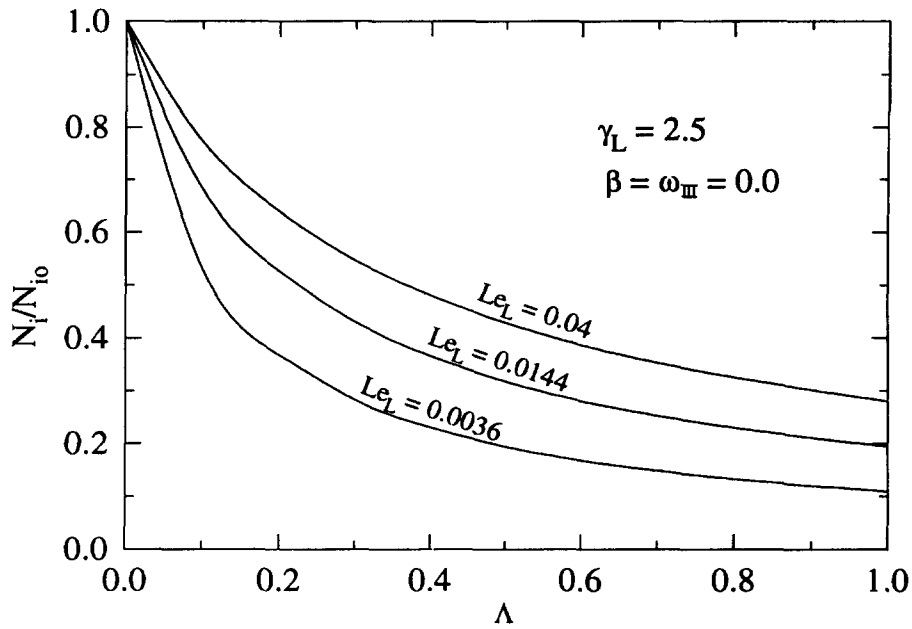


Fig. 4. Non-isothermal absorption in the absence of non-absorbables: reduced molal flux as a function of the dimensionless heat of absorption for different values of the liquid Lewis number ( $\gamma_L = 2.5$ ).

to the isothermal value  $N_{i0}$  at  $\Lambda = 0$  for all values of  $Le_L$ .

For most cases of practical interest,  $Le_L$  is much smaller than unity. For the low-flux cases where  $A$  is also small, equations (35) along with (27a) may be simplified to yield an explicit solution:

$$K_1 = \frac{\gamma_L \sqrt{Le_L}}{\Lambda + \gamma_L \sqrt{Le_L}} \quad (36a)$$

$$K_2 = \frac{\Lambda}{\Lambda + \gamma_L \sqrt{Le_L}} \quad (36b)$$

$$A = \frac{1}{\sqrt{\pi}} \frac{\sqrt{Le_L}}{\Lambda + \gamma_L \sqrt{Le_L}} \quad (36c)$$

In this case it is evident from equations (21) and (22) that the interfacial temperature and concentration are proportional to  $\Lambda$  and  $\gamma_L \sqrt{Le_L}$ , respectively. Before



beginning to analyze the dependence of the reduced mass flux ( $N_i/N_{i0}$ ) on the content of non-absorbables, an important observation may be made regarding its upper limit. It is evident from Fig. 1 that the absorption process, in the presence of non-absorbables, causes a decrease at the interface in the absorbate concentration on the gas side  $W_i$  with respect to its initial value  $W_\infty$ , while there is an increase in the concentration on the liquid side  $C_i$  with respect to  $C_\infty$ . The drop in  $W_i$  increases with  $N_i$  and the process must come to a halt before  $W_i$  vanishes.

4.3. Case 3

We will now consider the case of isothermal absorption in the presence of non-absorbables. The governing equations in effect here are equations (2) and (4) only, for the absorbate concentration  $C$  in the liquid and  $W$  in the gas phase. The boundary conditions (6b), (6d), (6e) and (6f) apply at the interface. The solution to equations (2) and (4) is given by equations (21) and (23) with the constants  $K_1$  and  $K_3$  calculated from a reduced version of (25b), as well as from (25d), (27a) and (27b), which at the same time yield the values of the constants  $A$  and  $B$ . In this case the absorption process is controlled by four out of the nine dimensionless parameters of the general case:  $\gamma_L$ ,  $P_e/P_o$ ,  $\omega_{III}$  and  $\phi$ . Figure 5 describes the interfacial concentrations on the liquid and gas sides ( $\gamma_i$  and  $\omega_i$ ) derived from equations (21) and (22) and Fig. 6 gives the dimensionless mass flux,  $N_i/N_{i0}$ , as functions of  $\phi$  for different values of  $\omega_{III}$  at typical values of  $\gamma_L = 2.5$  and  $P_e/P_o = 2.5$ . Note that  $\omega_i$  is a negative quantity while  $\gamma_i$  is positive, since on the gas side the absorbate concentration at the interface is lower than in the bulk, whereas on the liquid side the reverse is true, as

illustrated in Fig. 1. As evident, all three quantities,  $\omega_i$ ,  $\gamma_i$  and  $N_i/N_{i0}$ , remain approximately constant at small values of  $\phi$ , then decrease sharply as  $\phi$  increases.  $\phi$  is perhaps the most significant characteristic of non-absorbables, expressing the ratio of diffusion resistance in the gas to that in the liquid. In the absence of non-absorbables, diffusion resistance in the gas is nonexistent. With a small amount of non-absorbables present the mass flux is reduced considerably, the reduction being greater for greater  $\phi$ . From the definition of  $\phi$ , we note that a dense gas phase (large  $W_G$ ) makes for a smaller susceptibility to the harmful effects of non-absorbables; mixing in the gas phase near the interface due to local turbulence or natural convection may also be helpful in this respect. Figure 6 shows, as expected, that the larger the initial content of non-absorbables ( $\omega_{III}$ ) in the gas stream, the lower the mass flux.

From the set of equations governing this case it is possible to derive approximate analytical expressions for the two extreme conditions of very small and very large  $\phi$ . Consider first the condition of small  $\phi$ , small diffusive resistance in the gas phase. If  $A$  is of order unity, this makes  $B$  much smaller than unity, per equation (25d). Equation (27b), thus, reduces to  $K_3 = -B\sqrt{\pi}$  and substitution in the interfacial equilibrium condition (25b) yields:

$$\ln(1 - \omega_{III}) = [K_1(1 + \operatorname{erf} A) - 1] \ln \frac{P_e}{P_o} \quad (37)$$

From equations (37) and (27a),  $K_1$  and  $A$  can be found. In particular, for a small initial content of non-absorbables,  $\omega_{III} \ll 1$  and, hence,  $\ln(1 - \omega_{III}) \approx -\omega_{III}$ . In this case equation (37) reduces to the form:

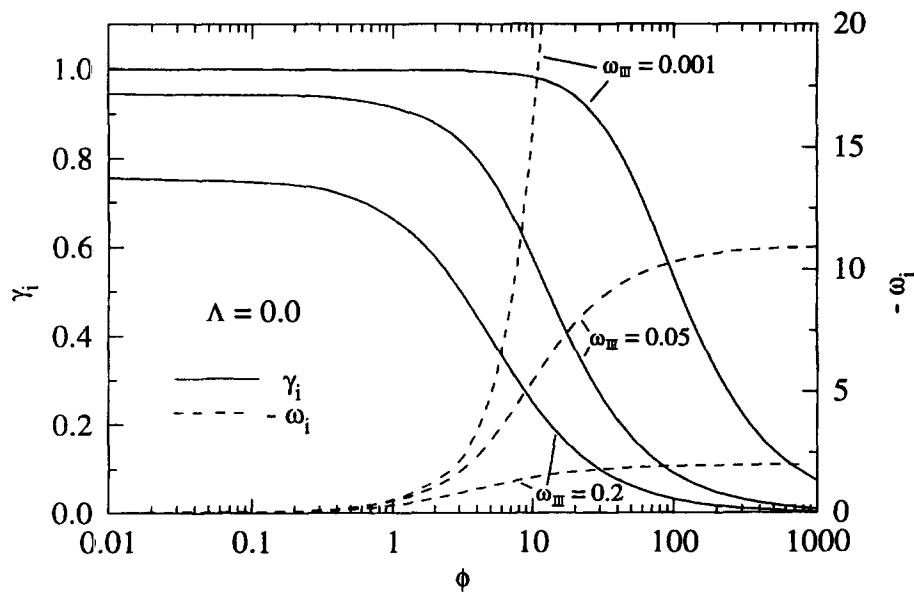


Fig. 5. Isothermal absorption in the presence of non-absorbables: interfacial concentrations in the liquid ( $\gamma_i$ ) and gas ( $\omega_i$ ) as functions of the diffusion resistance ratio  $\phi$ , for different mole fractions of non-absorbables ( $\gamma_L = 2.5$ ,  $P_e/P_o = 2.5$ ).

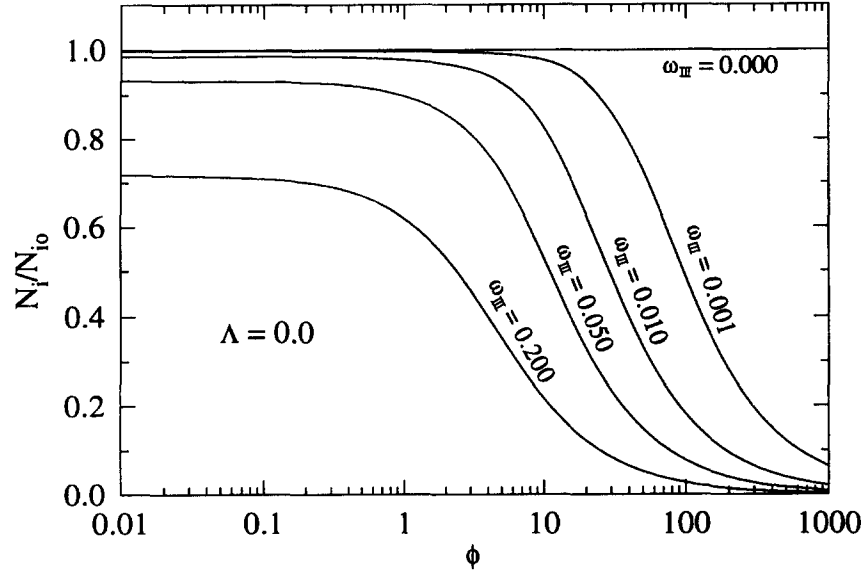


Fig. 6. Isothermal absorption in the presence of non-absorbables: reduced molal flux as a function of the diffusion resistance ratio  $\phi$ , for different mole fractions of non-absorbables ( $\gamma_L = 2.5$ ,  $P_e/P_o = 2.5$ ).

$$\frac{\omega_{III}}{\ln(P_e/P_o)} = 1 - K_1(1 + \operatorname{erf} A). \quad (38)$$

It is evident that as  $\omega_{III} \rightarrow 0$  this reduces to the already familiar problem of isothermal absorption in the absence of non-absorbables, as can be observed in Figs. 5 and 6.

Consider now the other extreme of large  $\phi$ , a large diffusive resistance in the gas phase.  $B$  is now large, per equation (25d) as long as  $\phi$  is large enough and  $A$  not too small. Equation (27b) may be rewritten as:

$$B\sqrt{\pi} \exp(B^2) = K_3 [B\sqrt{\pi}(1 - \operatorname{erf} B) \exp(B^2) - 1]. \quad (39)$$

Using an asymptotic expansion of the error function [29], the term in the brackets on the right-hand side of equation (39) reduces to  $-1/(2B^2)$  and, hence,  $K_3 = -2B^3\sqrt{\pi} \exp(B^2)$ . Clearly,  $K_3$  is very large and negative. Substitution in the interfacial equilibrium condition (25b) yields:

$$[K_1(1 + \operatorname{erf} A) - 1] \ln(P_e/P_o) = \ln(1 - 2B^2\omega_{III}) = \ln(1 - 2A^2\phi^2\omega_{III}). \quad (40)$$

From equations (40) and (27a),  $K_1$  and  $A$  can be found. In particular, if  $A$  is sufficiently small (as indicated by Fig. 6 for large  $\phi$ ), the term in the brackets on the left-hand side of equation (40) reduces to  $-1$  and we find:

$$A = \frac{1}{\phi\sqrt{2\omega_{III}}} \sqrt{1 - (P_o/P_e)}. \quad (41)$$

#### 4.4. Case 4

Having considered several particular cases of interest, we are now ready to study the behavior in the

general case of non-isothermal absorption in the presence of non-absorbables, influenced by nine characteristic dimensionless parameters. Three of these parameters have not yet appeared in any of the particular cases considered earlier:  $\theta_o$ ,  $Le_G$  and  $\beta$ . Figure 7 describes the reduced molal flux,  $N_i/N_{i0}$ , as a function of the dimensionless heat of absorption,  $\Lambda$ , for different mole fractions of non-absorbables,  $\omega_{III}$ . Typical values have been selected for the dimensionless parameters as indicated in the figure caption. It is evident that the heat of absorption as well as the presence of non-absorbables cause a reduction in the mass flux.

It was found that for most practical cases the parameters  $\theta_o$ ,  $\beta$  and  $Le_G$  have small effects on the absorption process. The parameter  $\beta$  describing the ratio of thermal properties between the gas and the liquid is almost always much smaller than unity and therefore negligible everywhere in equation (25c). Neglecting the terms with  $\beta$  is equivalent to having assumed *a priori* that heat transfer from the interface to the gas is negligible compared to that to the liquid. Figure 8 describes the effect of the pressure ratio  $P_e/P_o$  on the mass flux. The figure is drawn for typical values of the other parameters. Increasing the pressure ratio  $P_e/P_o$  causes an asymptotic increase in the mass flux, as may be expected.

## 5. CONCLUSION

The effect of non-absorbable gases on the simultaneous heat and mass transfer process in film absorption has been analyzed. The reduction in mass flux caused by the presence of these gases is most critically influenced by the diffusion resistance in the gas phase. When the latter is large compared to the resistance in the liquid, even minute quantities of non-

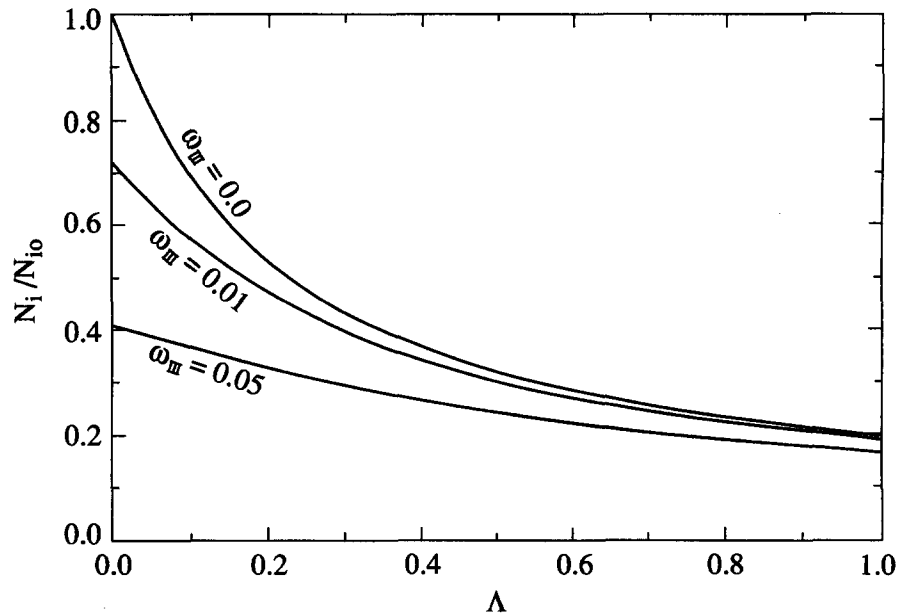


Fig. 7. Non-isothermal absorption in the presence of non-absorbables: reduced molal flux as a function of the dimensionless heat of absorption for different mole fractions of non-absorbables. In this typical case, the values of the dimensionless problem parameters are:  $\gamma_L = 2.5$ ,  $Le_L = 0.0144$ ,  $Le_G = 2.25$ ,  $\theta_o = 1.0$ ,  $\phi = 15.0$ ,  $\beta = 0.003$ ,  $P_e/P_o = 2.5$ .

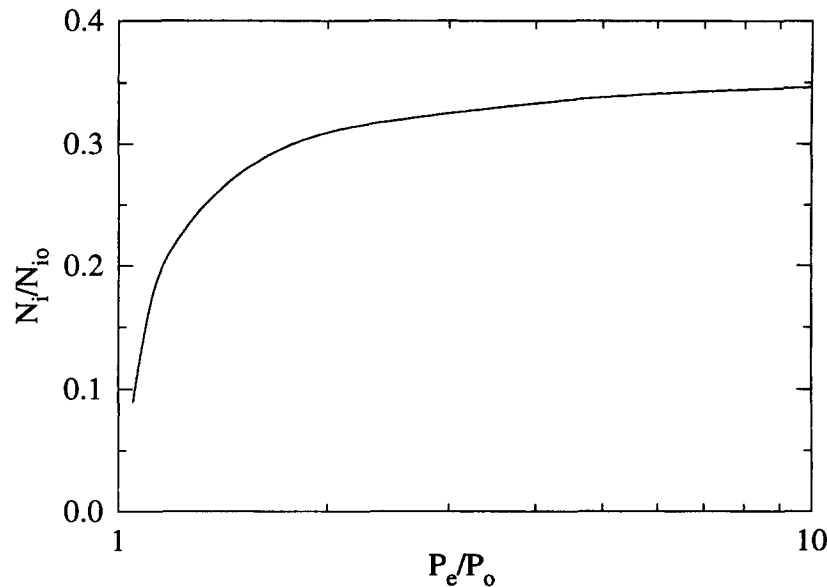


Fig. 8. Effect of pressure ratio ( $P_e/P_o$ ) on the reduced molal flux under typical values of the dimensionless problem parameters:  $\gamma_L = 2.5$ ,  $Le_L = 0.0144$ ,  $Le_G = 2.25$ ,  $\theta_o = 1.0$ ,  $\omega_{III} = 0.02$ ,  $\Lambda = 0.4$ ,  $\phi = 15.0$ ,  $\beta = 0.003$ .

absorbables can cause a considerable deterioration. The greater the initial mass fraction of non-absorbables, the lower the mass flux. The smaller the heat of absorption and the larger the pressure driving force—the greater the mass flux, as in the absence of non-absorbables.

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